MATHEMATICS APPLICATIONS

MAWA Semester 2 (Units 3 & 4) Examination 2019

Calculator-Assumed

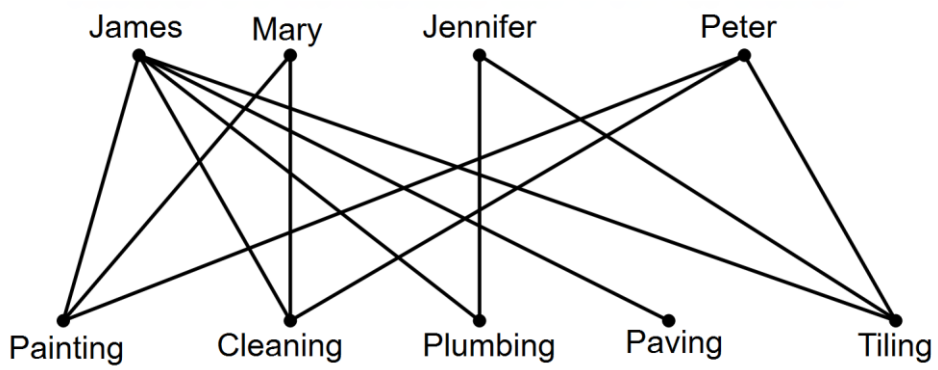
Marking Key

The release date for this exam and marking scheme is

- **the end of week 1 of term 4, 2019**

Section Two: Calculator-assumed (102 Marks)

Question 7 (a) (3 marks)

Solution


Marking key/mathematical behaviours	Marks
• labels Jennifer and Peter correctly	1
• identifies painting and tiling correctly	1
• draws the correct links for Peter	1

Question 7 (b) (3 marks)

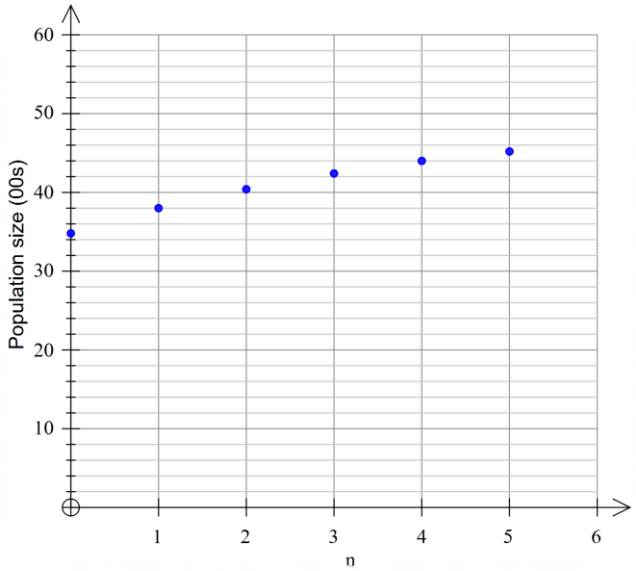
Solution
Bipartite graph The information given is in two separate groups or sets. People and tasks. The vertices represent the two groups and the edges represent the links or connections between the two groups. A bipartite graph makes this clear.

Marking key/mathematical behaviours	Marks
• gives correct type of graph	1
• identifies the two distinct sets as being represented by dots (or vertices)	1
• identifies the meaning of the edges.	1

Question 8 (a) **(5 marks)**

Solution

n	0	1	2	3	4	5
Population size (00's)	35	38	40.4	42.32	43.86	45.09



Marking key/mathematical behaviours	Marks
• completes at least two correct table values	1
• completes all table values correctly to 2 d.p.	1
• plots at least 4 points correctly	1
• plots all points correctly	1
• recognises discrete term values therefore no line connecting points	1

Question 8 (b) **(2 marks)**

Solution

Trend of population is increasing/growing, however, the growth rate is decreasing each time.

Marking key/mathematical behaviours	Marks
• recognises growth	1
• states growth rate is decreasing	1

Question 8 (c) **(3 marks)**

Solution

No, the relationship has a steady state solution of 5000, therefore never reaching the required 6000.

Marking key/mathematical behaviours	Marks
• states no	1
• identifies steady state solution	1
• correctly states value of steady state solution	1

Question 9 (a) **(2 marks)**

Solution	
$33722.67 \times r = 421.53$ $r = 0.012499899992$ New annual interest rate $= 12 \times 0.012499899992$ $= 15\%$	
Marking key/mathematical behaviours	Marks
• correctly determines $r = 0.012499899992$	1
• correctly converts answer to an annual interest rate	1

Question 9 (b) **(5 marks)**

Solution	
A = 1% of 38181.94 = 381.82 (2 d.p.) B = 1000 C = 38181.94 + 381.82 – 1000 = 37563.76 D = 12.5% of 829.8425 = 10.37 E = 829.84 + 10.37 = 840.21	
Marking key/mathematical behaviours	Marks
• correctly calculates 1% of 38181.94	1
• identifies constant repayment	1
• calculates amount owing at the end of fourth month	1
• correctly calculates 12.5% of 829.8425	1
• uses *D* with amount owing from Row *36* to calculate final payment	1

Question 9 (c) **(3 marks)**

Solution	
$T_{n+1} = 1.01T - 1000,$ $T_0 = 40000$	
Marking key/mathematical behaviours	Marks
• correctly writes the recursive rule (uses rate and repayment)	2
• correctly includes the initial term	1

Question 9 (d) **(2 marks)**

Solution	
Total repayments $= 11 \times 1000 + 25 \times 1500 + 840.21$ $= 49340.21$ Total interest paid $= 49340.21 - 40000$ $= 9340.21$	
Marking key/mathematical behaviours	Marks
• correctly calculates total amount repaid	1
• correctly determines total interest paid	1

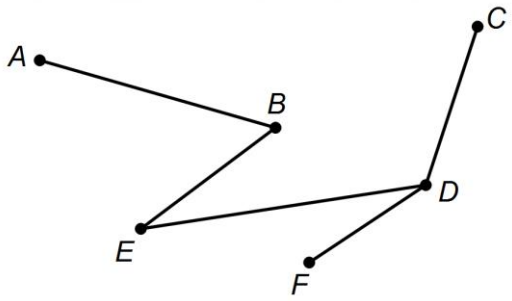
Question 10 **(5 marks)**

Solution	
Option 1: $I = 20000 \times 0.125 \times 10$ $= 25000$ Investment total $= 25000 + 20000$ $= 45000$ Option 2: $A = 20000(1 + \frac{10.75}{100(4)})^{10\times4}$ $= 57774.11$ Option 3: $A = 20000(1 + \frac{9.50}{100(52)})^{10\times52}$ $= 51669.39$ Therefore Option 2 is the best option to choose.	
Marking key/mathematical behaviours	Marks
• correctly calculates simple interest	1
• correctly calculates value of investment with principal and interest	1
• correctly calculates Option 2 using compound interest formula	1
• correctly calculates Option 3 using compound interest formula	1
• concludes that Option 2 is the best option based on calculations	1

Question 11 (a) **(2 marks)**

Solution

	A	B	C	D	E	F
A	~~-~~	~~8.5~~	~~12.5~~	~~-~~	~~8.7~~	~~-~~
B	8.5	~~-~~	~~7.6~~	~~6.2~~	~~4.5~~	~~-~~
C	~~12.5~~	~~7.6~~	~~-~~	6.4	~~-~~	~~8.8~~
D	~~-~~	~~6.2~~	~~6.4~~	~~-~~	5.8	~~3.1~~
E	8.7	4.5	~~-~~	~~5.8~~	~~-~~	~~6.8~~
F	~~-~~	~~-~~	~~8.8~~	3.1	~~6.8~~	~~-~~



Marking key/mathematical behaviours	Marks
• correctly draws at least 3 edges	1
• correctly draws minimum spanning tree	1

Question 11 (b) **(2 marks)**

Solution
$Distance = 8.5 + 4.5 + 5.8 + 6.4 + 3.1$ $= 28.3m$ $Cost = 28.3 \times 840$ $= 23772$

Marking key/mathematical behaviours	Marks
• calculates correct total distance of minimum spanning tree	1
• calculates correct minimum cost of project	1

Question 11 (c) **(2 marks)**

Solution
The minimum spanning tree would no longer include edge *DC*, and would have *BC* instead. This also changes total minimum cost. The total distance is increased by 1.2 metres at an extra cost of $1008.

Marking key/mathematical behaviours	Marks
• identifies correct edge change	1
• states correct increase of cost	1

Question 12 (a) **(2 marks)**

Solution	
A 7-point average would be the most appropriate, reflecting a weekly cycle.	
Marking key/mathematical behaviours	Marks
• indicates a 7-point average	1
• makes the link with a weekly cycle	1

Question 12 (b) **(2 marks)**

Solution	
Trend appears to be gradually rising over time as reflected by rising minimum points in each cycle. The rise in the peaks of each cycle are not so clear but day 16 is higher that day 3, even if day 9 has a slight dip.	
Marking key/mathematical behaviours	Marks
• indicates a rising trend	1
• gives a plausible explanation of how this has been decided	1

Question 12 (c) **(4 marks)**

Solution

Day number	4	5	6	7	8	9	10	11	12	13	14
Price/litre	127.5	125.9	121.0	130.4	131.0	131.1	130.4	129.0	126.9	123.8	131.5

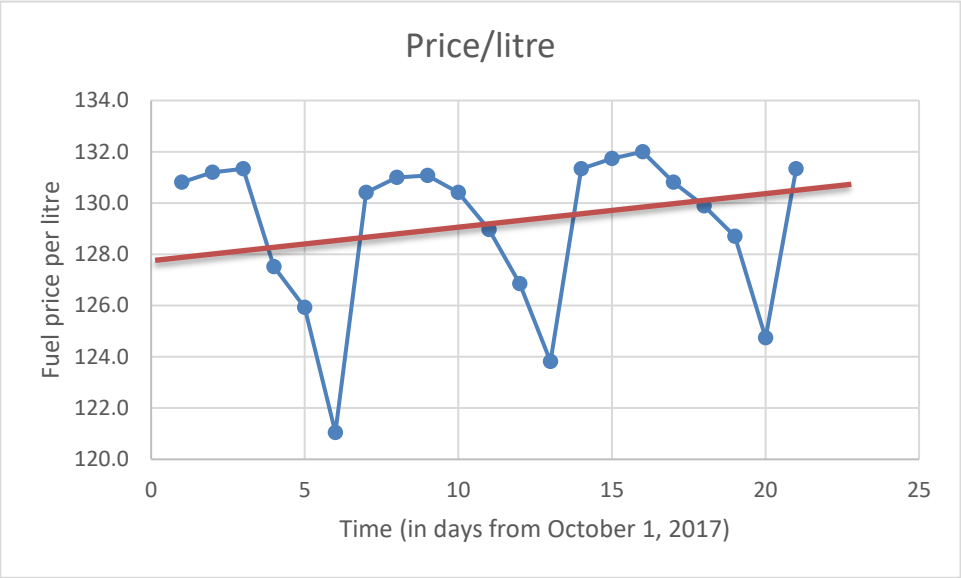
The weekly mean for week 2, will be the average of the price for the 7 days as highlighted.

The percentage of the weekly mean for day 9 will be $\frac{131.1}{129.1} \times 100 = 101.55\%$

The percentage of the weekly mean for day 10 will be $\frac{130.4}{129.1} \times 100 = 101.01\%$

Marking key/mathematical behaviours	Marks
• identifies the need to calculate the mean for week 2 (days 8-14)	1
• calculates the weekly mean as 129.1 cents	1
• calculates the correct % weekly mean for day 9 (to at least 1 d.p.)	1
• calculates the correct % weekly mean for day 10 (to at least 1 d.p.)	1

Question 12 (d) **(3 marks)**

Solution


Marking key/mathematical behaviours	Marks
• identifies the y-intercept at ~129.3	1
• identifies at least one other point, eg (18,130.34)	1
• draws the appropriate line	1

Question 12 (e) **(2 marks)**

Solution
At $d = 25, P = 0.1339 * 25 + 127.93 = 131.28$ Given that the seasonal index for Wednesday is 0.9961 The actual price per litre for day 25 $= 131.28 \times 0.9961 = 130.77$

Marking key/mathematical behaviours	Marks
• substitutes $d = 25$ into the least-squares regression line for the moving avaerages.	1
• applies the Wednesday index to calculate the actual value	1

Question 12 (f) **(2 marks)**

Solution
The prediction is reliable as it is in the next cycle immediately after the end of the data used to calculate the least-squares regression line for the moving averages (4 days past the known data – but with the one cycle).

Marking key/mathematical behaviours	Marks
• recognises prediction is reliable	1
• recognises it is within the next cycle, even though 4 days out.	1

Question 13 (a) **(4 marks)**

Solution	
$BACDM = 60$ $BACEFGM = 20$ $BECFGM = 70$ $BEFGM = 30$ $BEHGM = 20$ $BHGM = 20$ $BHM = 180$ Total max flow $= 60 + 20 + 70 + 30 + 20 + 20 + 180$ $= 400\,ppl/\min$	
Marking key/mathematical behaviours	Marks
• evidence of systematic approach	1
• correctly lists at least 3 routes and corresponding flow	1
• correctly lists all possible routes and corresponding flow	1
• correctly states maximum flow of network	1

Question 13 (b) **(3 marks)**

Solution	
Cut $I = 480$ Cut $II = 410$ Cut $III = 400$ Therefore the cut that supports max. flow of 400ppl/min is Cut III	
Marking key/mathematical behaviours	Marks
• correctly calculate at least 2 cuts	1
• correctly calculate all three cut totals	1
• states Cut III	1

Question 13 (c) **(3 marks)**

Solution	
The upgrade will have no effect on maximum flow. Lift location edge *EH* has not been entirely used and lift location *EF* has no flow leading in or out.	
Marking key/mathematical behaviours	Marks
• identifies lift locations	1
• states no effect on max flow	1
• provides correct reason	1

Question 14 (a) **(4 marks)**

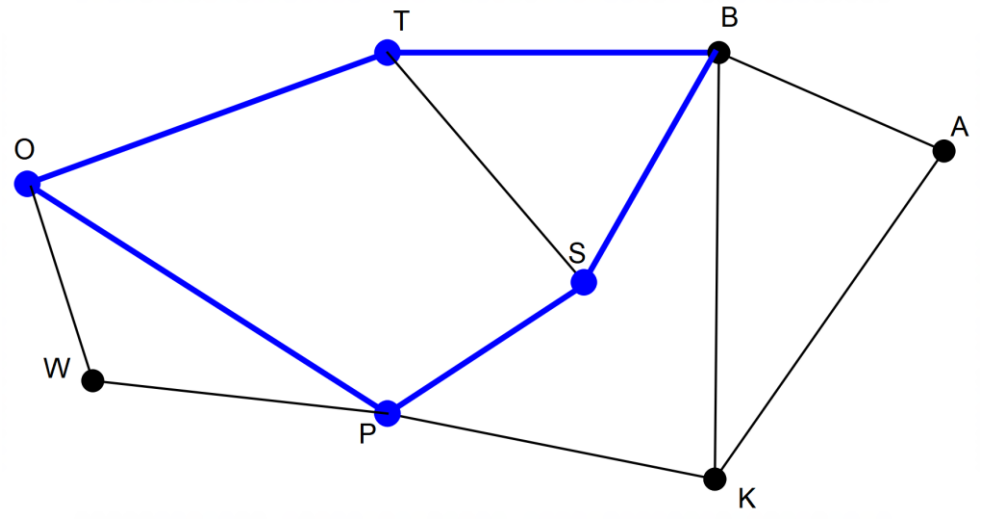
Solution	
$v = 8$, $f = 6$, $e = 12$ and $v + f - 2 = e$ i.e. $8 + 6 - 2 = 12$	
Marking key/mathematical behaviours	Marks
• identifies the number of faces, edges and vertices	3
• verifies Euler's rule applies	1

Question 14 (b) **(2 marks)**

Solution	
No, it is not a Eulerian graph. A Eulerian graph is connected and traversable, starting and finishing at the same vertex. It must have no odd vertices. The given graph has 4 odd vertices (O, T, S and K).	
Marking key/mathematical behaviours	Marks
• identifies the graph as non-Eulerian.	1
• Indicates a correct reason – i.e. it has 4 odd vertices	1

Question 14 (c) **(3 marks)**

Solution



The specified walk defines a closed circuit which is also a Hamiltonian cycle. The walk does not have any repeated vertices or edges and starts & ends at the same vertex

Marking key/mathematical behaviours	Marks
• identifies the required walk	1
• indicates a closed circuit/Hamiltonian cycle	1
• identifies characteristics	1

Question 15 (a) **(3 marks)**

Solution	
$r = 0.5228$ Therefore relationship is weak, positive, linear.	
Marking key/mathematical behaviours	Marks
• calculates correlation coefficient correctly to 4 d.p. • states there is a weak positive linear relationship between variables	1 2

Question 15 (b) **(2 marks)**

Solution	
Explanatory variable = height Response variable = mass	
Marking key/mathematical behaviours	Marks
• correctly identifies explanatory variable • correctly identifies response variable	1 1

Question 15 (c) **(2 marks)**

Solution	
$\hat{m} = 0.6795h - 42.3567$	
Marking key/mathematical behaviours	Marks
• correctly determines gradient • correctly determines y-intercept	1 1

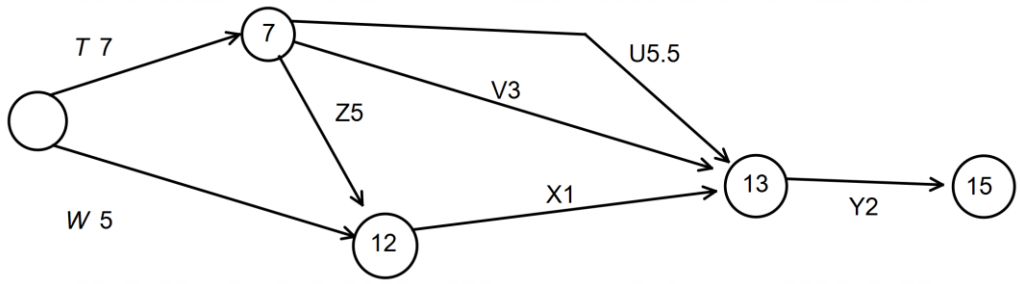
Question 15 (d) **(3 marks)**

Solution	
$r^2_{hm} = 0.2734$ This value suggests that 27.34% of the change in mass can be attributed to the change in height. Since 27.34% is a low percentage, a linear model may not be appropriate.	
Marking key/mathematical behaviours	Marks
• correctly calculates coefficient of determination • correctly explain significance of the coefficient of determination • clearly states that a linear model may not be appropriate for these data	1 1 1

Question 15 (e) **(4 marks)**

Solution	
$h = 182 : \hat{m} = 0.6795(182) - 42.3567 = 81.31kg$ Prediction is reliable as it involves interpolation $h = 200 : \hat{m} = 0.6795(200) - 42.3567 = 93.54kg$ Prediction is unreliable as it involves extrapolation	
Marking key/mathematical behaviours	Marks
• correctly calculates predicted value for h = 182	1
• concludes prediction for h = 182 is reliable due to interpolation	1
• correctly calculates predicted value for h = 200	1
• concludes prediction for h = 200 is unreliable due to extrapolation	1

Question 16 (a) **(3 marks)**

Solution	
	
Marking key/mathematical behaviours	Marks
• has U, V and Z coming out from T	1
• completes network with correct edges	1
• edges have correct labels (task and duration)	1

Question 16 (b) **(2 marks)**

Solution	
Critical path is $TZXY$ Minimum completion time is 15 days	
Marking key/mathematical behaviours	Marks
• states the correct critical path	2
• calculates correct minimum completion time	2

Question 16 (c) **(2 marks)**

Solution	
(i) float time for task Z = 0 as on critical path (ii) task W has a float time of 7 days, so the latest start date is 7 days (1 week)	
Marking key/mathematical behaviours	Marks
• correctly states there is no float time for Task Z	1
• correctly determines latest start time	1

Question 16 (d) **(3 marks)**

Solution	
Critical path is now TVY and the minimum completion time has been increased by 2 days	
Marking key/mathematical behaviours	Marks
• correctly states new critical path	1
• identifies increase in completion time	1
• states increase of 2 days	1

Question 17 (a) **(2 marks)**

Solution	
Third year $= 30000 \times 1.03^2 = 31827$ Fourth year $30000 \times 1.03^3 = 32781.81$	
Marking key/mathematical behaviours	Marks
• calculates the amount withdrawn in third year • calculates the amount withdrawn in fourth year	1 1

Question 17 (b) **(2 marks)**

Solution	
Using a spreadsheet or sequence function (or otherwise), the amount left in the annuity after the 10th withdrawal is \$244090.48	
Marking key/mathematical behaviours	Marks
• uses an appropriate method • correctly calculates the amount remaining after the 10th withdrawal	1 1

Question 17 (c) **(2 marks)**

Solution	
Using a spreadsheet or sequence function (or otherwise), it will take 17 years for the annuity to reach a balance of 0 (after 16 withdrawals there is only 5897.61 remaining)	
Marking key/mathematical behaviours	Marks
• correctly calculates the number of years for the annuity to reach a balance of 0.	2